PRACTICE FINAL - MATH 20C - FALL 2020

Problem 1. (3 points) Calculate the distance between the plane spanned by $a = (1, 1, 0)$, $b = (0, 2, 1)$, $c = (2, 0, 3)$ and the point $p = (2, 3, 4)$ via the following steps.

(1) Find a normal vector of the plane by taking the cross product $(a-b)\times(a-c)$ or $(b-a)\times(b-c)$ or $(c-a)\times(c-b)$.

(2) Find the length of the projection of $p-a$ or $p-b$ or $p-c$ onto the normal vector you found in part (1).

Problem 2.(Challenge) Using the geometric meaning of the cross product, dot product, and determinant, prove the triple product formula, i.e. that

$$|(a \times b) \cdot c| = |\det([a, b, c])|$$

where $[a, b, c]$ is the matrix with vector a in the first row, b in the second row and c in the third row. Recall that $|a \times b| = ||a||||b||\sin(\theta)$ where θ is the angle between a and b, $u \cdot v = ||u||||v||\cos(\gamma)$ where γ is the angle between u and v, and $|\det([a, b, c])| =$ the volume of the parallelepiped spanned by the vectors a, b, and c.

Problem 3. Write down the parametrization of the line perpendicular to the grahp of the function $f(x, y) = 4 - x^2 - y^2$ at the point $(1, 1, 2)$. (Hint: use tha tangent plane to the graph of the function).

Problem 4. Find all the point (x, y) where the tangent plane to the graph of the function $f(x, y) = x^2 - 4x + y^2 - 2y + 6$ is parallel to the plane $x + 2y - z = 3$.

Problem 5. Consider the function $f(x, y) = x^2 + y^2 - y$. Find the global maxima and minima of f on the region

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1,\ y \geq 0\}.$$

Problem 6. Let x, y, and z be angles of a triangle measured in radians between 0 and π (Hint: What constraint does this put on the possible angles?). What is the maximum value of

$$x + \frac{y^2}{2} + \frac{z^3}{3}?$$

Problem 7. Find the arc length of the path

$$\overline{c}(t) = (t, t\sin(t), t\cos(t))$$

for $0 \leq t \leq 1$.

Problem 8. Evaluate the integral:

$$\int_0^4 \int_{y/2}^2 e^{x^2}\, dx dy$$

by changing the order of integration.